Tax, Surplus, and Deadweight Loss

Given the supply and demand curves for butter, $P = 4Q$ and $P = 100 - Q$ respectively, with quantities expressed in kg.

1. Find the equilibrium price and quantity and plot the graph.
2. The government imposes a \$10 tax per kg of consumed butter. Find the new equilibrium and plot the demand curve "once the consumer has paid the tax." Calculate the tax revenue and the consumer and producer surplus.
3. Calculate the tax burden on suppliers and the tax burden on demanders (solve this exercise using the normal method for calculating elasticity). Also, calculate the tax's efficiency loss.

Solutions

1. To find the equilibrium price and quantity, equate the equations:

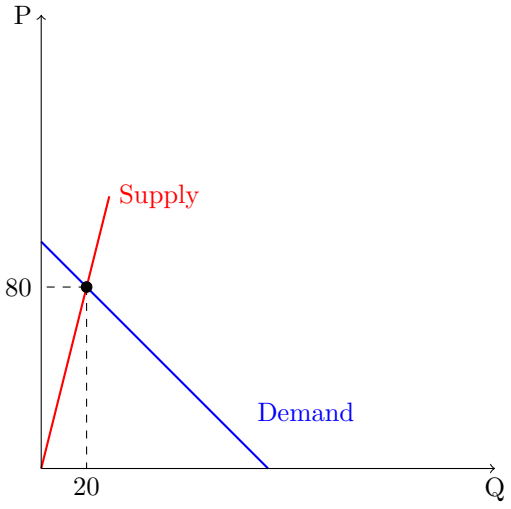
$$4Q = 100 - Q$$

Solving for Q:

$$5Q = 100$$

$$Q = 20$$

Replacing this value in the supply or demand equation, we obtain the equilibrium price: $P = 80$.



2. As the government imposes a tax, the price paid by consumers is no longer equal to the price received by producers. The price received by the producer is lower since part of it corresponds to the tax: $P_o = p_d - T$. Where $T = 10$. Then $P_o = 100 - Q - 10$, which is: $P_o = 90 - Q$. Replacing P_0 in this last equation.

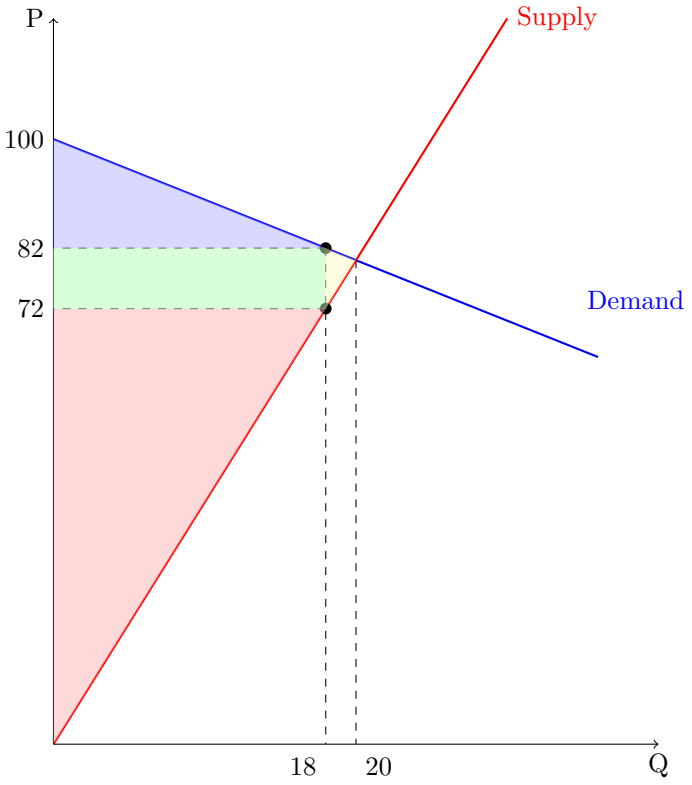
$$4Q = 90 - Q$$

$$5Q = 90$$

$$Q = 18$$

This is the new equilibrium quantity. Now, there is not only one equilibrium price, but we have both consumer and producer prices. The consumer price is $P_d = 100 - Q = 82$. Meanwhile, the producer price: $P_o = P_d - 10 = 82 - 10 = 72$. Notice that the producer receives less than what the consumer pays; this is because part of the income from the sale goes to the government.

3. Let's start with the efficiency loss, which is represented by the yellow triangle in the previous graph:



The blue area is the consumer surplus, the red area is the producer surplus, the green rectangle is the tax revenue, and the small yellow triangle is the efficiency loss, deadweight loss, or irrecoverable loss. To calculate the surpluses, we use the base times height divided by 2 rule.

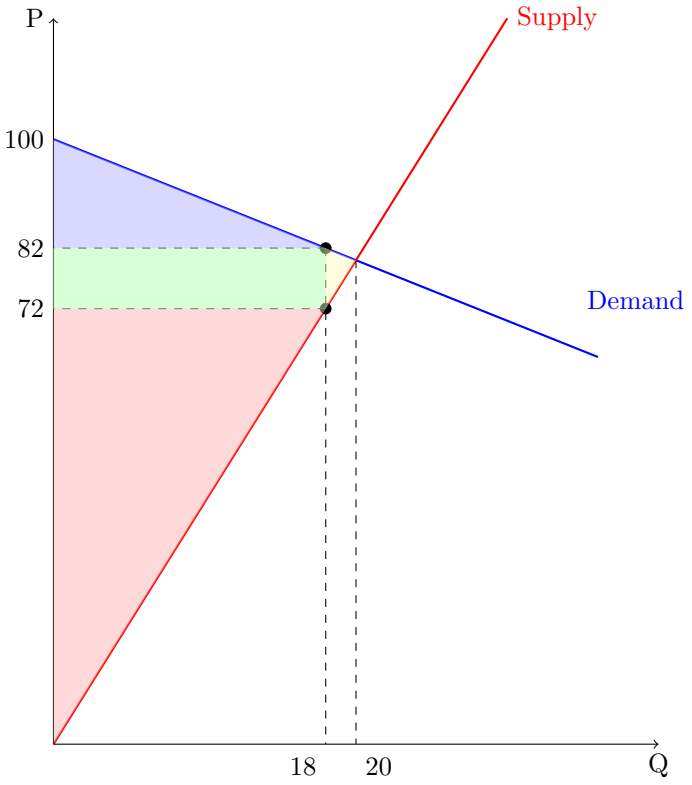
$$\text{Producer surplus: } \frac{7218}{2} = 648$$

$$\text{Consumer surplus: } \frac{1218}{2} = 162$$

For the tax revenue, we can think of it as the area of the rectangle or as the equilibrium quantity traded multiplied by the tax (which is 10 in this case)

$$\text{Tax revenue: } 1018 = 180$$

4. Let's start with the efficiency loss, which is represented by the yellow triangle in the previous graph:



The result is the area of a triangle, so:

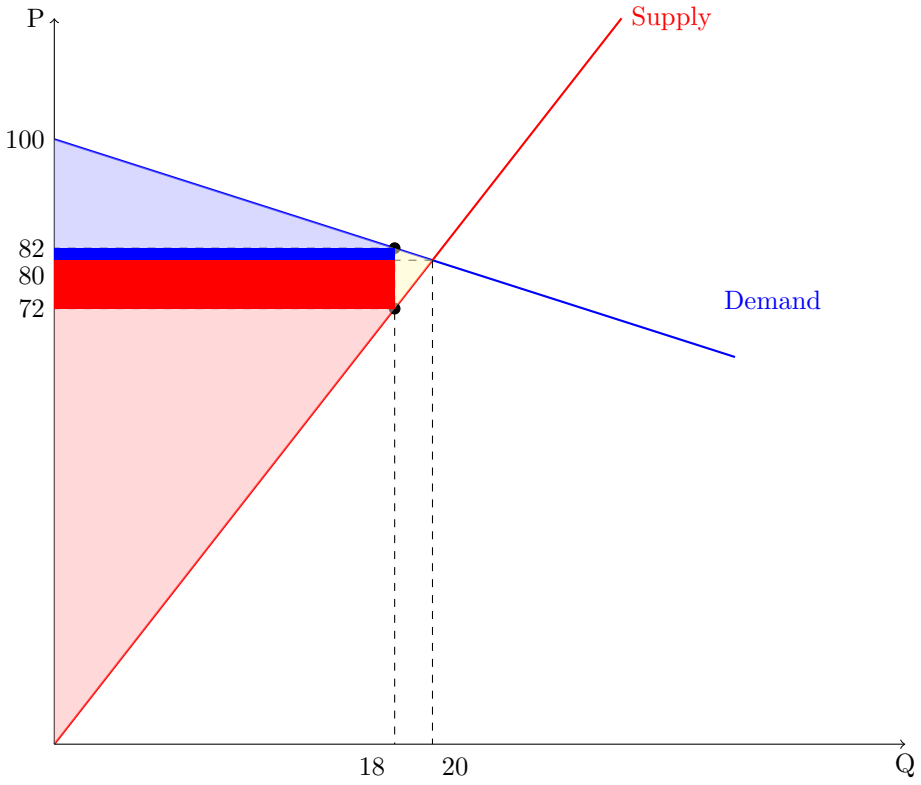
$$\text{Deadweight loss: } \frac{210}{2} = 10$$

To determine who bears the greater burden of the tax, it is not relevant whether the tax is on producers or consumers, but rather the relevant factor is the price elasticities of supply and demand. If the price elasticity of demand is greater than the price elasticity of supply, the tax affects suppliers more. If the price elasticity of supply is greater than the price elasticity of demand, the tax affects consumers more. Our functions expressed with Q isolated are: $Q_d = 100 - P$ and $Q_o = \frac{P}{4}$. Using the formulas for elasticities to calculate with functions:

$$\text{Price elasticity of demand: } |\frac{\Delta Q}{\Delta P}\frac{P^*}{Q^*}| = |-1 * \frac{80}{20}| = 4$$

$$\text{Price elasticity of supply: } |\frac{\Delta Q}{\Delta P}\frac{P^*}{Q^*}| = |\frac{1}{4}\frac{80}{20}| = 1$$

Remember that $\frac{\Delta Q}{\Delta P}$ is equal to the slope of the demand curve or the supply curve, respectively. Since $4 > 1$, the burden will fall more on the producers. We can see this graphically.



The incidence of the tax is the revenue area divided by the horizontal line generated by the equilibrium price (without the tax). The calculation of these two areas is:

$$\text{Burden on consumers: } 218 = 36$$

$$\text{Burden on producers: } 818 = 144$$

This shows us that the tax revenue is generated on one hand by the loss of consumer surplus and on the other hand by the loss of producer surplus. However, the irrecoverable loss or deadweight loss is a loss of surplus that does not turn into revenue.